"Can you do Addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?"

"I don't know," said Alice. "I lost count."

"She can't do Addition," the Red queen interrupted. "Can you do subtraction? Take nine from eight."

"Nine from eight I can't, you know," Alice replied very readily: "but—"

"She can't do Subtraction," said the White Queen. "Can you do Division? Divide a loaf by a knife—what's the answer to *that*?"

"I suppose—"Alice was beginning, but the Red Queen answered for her. "Bread-and-butter, of course. Try another Subtraction sum. Take a bone from a dog: what remains?"

Alice considered. "The bone wouldn't remain, of course, if I took it—and the dog wouldn't remain: it would come to bite me—and then I'm sure *I* shouldn't remain!"

"Then you think nothing would remain?" said the Red Queen.

"I think that's the answer."

"Wrong, as usual," said the Red Queen: "the dog's temper would remain."

"But I don't see how—"

"Why look here!" the Red Queen cried. "The dog would lose its temper, wouldn't it?"

"Perhaps it would," Alice replied cautiously.

"Then if the dog went away, its temper would remain!" the Queen exclaimed triumphantly.

Alice said, as gravely as she could, "They might go different ways." But she couldn't help thinking to herself "What dreadful nonsense we *are* talking!"

from *Through the Looking-Glass and What Alice Found There*, by Lewis Carroll

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## Sermon: To Infinity And Beyond

According to history, or perhaps legend, there once lived a man named Hippasus of Metapontum, who had a problem. Hippasus was a member of a sect, a sect founded by Pythagoras of Samos and remembered better by us as the Pythagoreans. As a member of that ancient secret society, he had been privy to the deep and wonderful truths, mystical truths in their eyes, which their research had revealed. And now he had made another such discovery, a truly astounding one. It was a discovery that was very unnerving—to a Pythagorean, at least—because it threatened everything they held most sacred.

Now, of course, the research in which the Pythagoreans engaged was not done through controlled experiment or carefully recorded observation but, rather, through pure intellectual effort by being the first, some 2500 years ago, to use mathematical proof. We can only imagine how intoxicating this technique could be, to be able to make useful, interesting, reliable, and universal discoveries about the nature of the world through reason alone. Indeed, learning of it about a century later, it might have caused the traveler Plato to abandon the intellectual agnosticism of his revered teacher, Socrates, and replace it with a new faith in the power of reason to uncover truths which did not depend on human situations or points of view but were instead external and essential to all points of view and foundational to reality itself.

While this is speculative in regards to Plato, we do know it had a similar effect on its first users. Presumably because of their successes with geometry and arithmetic, the Pythagoreans regarded the triangle and counting numbers (like 1, 2, and 3), the most fundamental concepts of their studies, to be the most basic elements of the Universe. To them, triangles and numbers were divinely perfect. And therein was the source of Hippasus's problem.

Let me pause here to give a brief—really, the briefest—review of some geometry. All you have to remember is what a right-angled triangle is, one in which one of the corners is a perfect "L-shape", and that the side opposite that "L-shaped" corner is called the hypotenuse. There, that's all.

What Hippasus discovered was this: that the length of the hypotenuse of many right-angled triangles cannot be represented by counting numbers.

Oh, you probably knew that anyway. According to the most famous law of geometry, the one Pythagoras himself supposedly discovered, a right-angled triangle with two sides that are one unit in length has to have a hypotenuse that is $\sqrt{2}$ units in length. But what Hippasus proved is that $\sqrt{2}$ is not equal in value to any fraction. That is, it cannot be calculated by dividing one counting number by another. $\sqrt{2}$ cannot be represented as a ratio of two counting numbers. $\sqrt{2}$ is, in the technical sense, *irrational*.

Big deal! But think of what this meant to the Pythagoreans. It meant that their two most holy entities, the triangle and the counting number, were in conflict, insufficient to represent the world. They were flawed. Hippasus had used their sacred technique to show them the inadequacy of their own theology.

This did not go over well with the Pythagoreans. There are multiple versions of the resolution of this story. Hippasus was banished from the sect, or he was thrown by his peers from aboard the very ship upon which he made the discovery, or Pythagoras himself strangled him, but all to keep this dangerous secret from becoming known to outsiders and protect the world from this heresy.

And it is here that this example from the history of mathematics becomes both familiar and instructive to us as Unitarian-Universalists. We know this kind of story, the punished non-conformist, the murdered heretic, all too well. But in many of these cases we think of them as victims both because they might have been right and because they are individuals of inherent worth regardless of the truth of their beliefs. We sympathize with them not only because their views might have been enlightened but also because we feel sincere beliefs require respect even if they are in error. After all, loveable and quirky non-conformists and inspiring and passionate heretics might still be wrong. Because of this complication, most of these cases will not serve to illustrate the point I will try to make: that commitment to rationality is essential to our religious perspective.

So I am choosing examples from mathematics, a world in which the heretics are usually right and in which their reactionary persecutors not only disagreed with the dissenter but seemingly abandoned their commitment to reasoned inquiry in so doing. I borrow some of these examples from the book *The Mystery of the Aleph: Mathematics, the Kabbalah, and the Search for Infinity*, by Amir Aczel, and I mean for these examples to show that we, who already are inclined to prefer reason over tradition, authority, and revelation, could do well to maintain our trust in it even when it takes us on strange journeys. If you like, you could say that I am proposing a paradoxical thesis: that faith in rationality is a good thing.

First, some definitions: By reason I mean logic—the basic patterns by which truths are connected—but also I mean judgment, the faculty by which we obtain our starting truths through observation and intuition. By rationality I mean a system of reasoning principles in logic and judgment that we, as thinking creatures, can use in order to come to agreement about truth. And by commitment to rationality I mean embracing the belief that there is such a system.

At this point I would like you to take special note of what Hippasus has done for us. The very word “rationality” stems from those “ratios” that the Pythagoreans held so dear. To them, ratios–proportions –made such perfect sense that they equated any explanation related to ratios, any “rational” explanation, to clear thinking. But Hippasus showed through his clear thinking that we could get to concepts that transcended ratios. Hence, what is now “rational” is no longer just “ratio-nal”. Get it? In a very strong sense he redefined what clear thought was. What was once inscrutable paradox has become unambiguous axiom

Hippasus might have given his life for this insight, and all because his colleagues who claimed to value rationality really only valued ratios. They chose not to wrestle with the implications for their worldview. For them it was “turtles all the way down.” I suppose Hippasus could have destroyed his proof when he first realized what it meant, but he did not, and, although his faith in his colleagues proved misplaced, his trust in the ultimate accessibility of his proof—his commitment to rationality, was not. We take it for granted now: not all values are fractions. And it is in the commonplaceness of this notion that we find the evidence of Hippasus' victory. His argument succeeded not only in that it was right but also in that it got buy-in from others even after his demise. This is the true worth of rationality: it persuades, sometimes very slowly and across generations, by appealing to deep intellectual functions we thinkers all share. In this way it works to sweep away ignorance, misconception, and prejudice. It is, if not the quickest tool for this, an irresistible one.

I'll offer another example from the history of Mathematics, in this case from a period a little closer to us and to which we might relate a little better. I am thinking of Georg <GAY-org> Cantor, the 19th century German who also, by inspiration and hard work, uncovered an astounding mathematical principle.

Cantor's interest was in the properties of infinite sets, like, for example, the counting numbers 1, 2, 3, … and so on. As we learn at a young age, no matter how far you count, you can't run out of numbers. There is no limit to them, so we call the number of numbers infinite. You might think that was the end of the story but, no, there is much more to find out. You can, for example, prove that the number of fractions–those rational numbers that so bothered the Pythagoreans– is the same as the number of counting numbers. It might seem as if there are many more fractions than counting numbers (what about all those fractions between 1 and 2, for example) but, in fact, we can come up with a scheme in which you can show that all the fractions can be counted – there are enough counting numbers to give one to every fraction.

As counter-intuitive as this might seem, Cantor proved something much more paradoxical. In 1873, by a clever proof that also involved lists, he showed that when you count the irrational numbers, like √2, you run out of numbers before you run out of irrationals. Now, remember, we are talking about two infinite sets here: the set of all counting numbers and the set of all irrational numbers. What he showed was that one infinite set is bigger than another. Moreover he devised a technique by which an endless supply of bigger infinite sets could be derived from "smaller" ones. As a contemporary and peer of said when Cantor shared the proof with him: “I see it, but I don’t believe it.” Cantor really had gone to infinity and beyond.

His discovery was controversial and was received with hostility in many places. Unlike Hippasus, Cantor was not murdered but he did experience a kind of banishment because of his ideas. There were many who saw peril for the foundations of mathematics in this result and refused to accept it, objecting that the notion of multiple infinite sets moved mathematics into a sordid fantasy realm. Indeed one of Cantor’s most vociferous opponents, Leopold Kronecker, called Cantor a “corrupter of youth” for teaching about such chimerical nonsense when he should have been teaching sound, sober mathematics. Kronecker, situated at the University of Berlin, the leading school in mathematics at that time, also referred to Cantor as a “renegade” and “charlatan.” He used his status to pressure journals of the time into rejecting Cantor’s papers, and was largely responsible for blocking Cantor’s appointment to the Berlin faculty. There were also theological attacks against Cantor, with charges that Cantor’s proof either amounted to an attack on the infinite nature of God or, because it held to multiple infinities, was equivalent to pantheism. It is a romantic notion to suggest that such criticism led to the mental illness, now attributed to bipolarity, that Cantor experienced for the last thirty years of his life, but surely it did not help.

It seems that even among mathematicians, as the Red Queen says, the temper always remains, especially when we, like Alice, lose count! What appears to happen to human beings, even those who practice rational inquiry every day, is when reason takes them into strange and unexpected places they cede their faith in it in deference to comforting commitments of other sorts such as those that arise out of theology or familiarity or just plain self-interest.

This tendency in human nature is probably why the $18^{th}$ century British philosopher David Hume contended that reason will always be a loser in such battles:

> It is obvious that when we have the prospect of pain or pleasure, we feel a consequent emotion of aversion or propensity and are carried to avoid or embrace that which will give us uneasiness or satisfaction. Here reasoning takes place to discover the relation of cause and effect and subsequently guides our actions, but our impulses do not arise from reason. Neither can reason prevent any volition. Reason is the slave of the passions and can never pretend to any other office than to serve and obey them.

And some interpretation of this is very much alive today. It is considered idealistic, even naïve to expect to influence the beliefs and actions of any individual or group by appeal to reason. Carrots and sticks, sub-rational gut-level appeals, subliminal persuasion—that's what motivates. Hence we get sound bites and film clips and fist bumps.

However, as much as I admire Hume, I think he is wrong on this point and his cynicism about rational discourse too pessimistic. There are two reasons I think so. First, and I think even Hume would admit this; it is not as if our impulses, our passions, are always neatly atomized components of our nature. They are sometimes in conflict—should I eat or sleep now?—and sometimes confused. Appeals to rationality, both in conversation and introspection, can help sort this out and thereby can rationality play a primary role in our decisions.

Second, and I think Hume does not see this possibility, just as we have emotional reactions to all kinds of pains and pleasures, I think on a basic level, we find clear thinking beautiful...pleasurable...compelling, too. To the mind, rationality is not just a means to an end but an end in itself, to be pursued for its own sake as well as its usefulness. In this way, too, it becomes a part of those things which provide us with initial motivation and not just a plan of action. What I am really trying to poke at here is that old Star Trek idea that reason and emotion are cleanly bifurcated in our natures, Mr. Spock on one side, Dr. McCoy on the other. Instead I propose to you that they interact in complicated ways and not as one subordinate to the other. In fact, they may be, at bottom, the very same kinds of things. There is no reason, then, to expect rational appeals will always lose to emotional ones.

So, for we who share a liberal and open-ended religious perspective, relying on insights from many different sources and traditions, here is what a commitment to rationality gets us:

- A means of navigating through apparently contradictory viewpoints, allowing us to see the wisdom in paradox

- A tool for building a shared set of beliefs among all peoples by appeal to their common faculties for seeing the truth and a reason to hope that shared beliefs among human beings are not only possible, but inevitable
- A check against rash and impulsive rejections of new but threatening ideas that we all, at times, can be guilty of.
- An innately motivating methodology that satisfies our intellectual passions in a deep way.

In short, it provides us with measures of wisdom, hope, prudence, and passion. And for those who are not familiar with our approach, who see at first a blooming, buzzing confusion of ideas, we have to share with them our fondness for rationality and thereby our deep faith that, no matter how many kinds of ideas we consider, jointly or alone, in the end everyone one of us can work it out for ourselves and help each other to work through it, too

May it be so, and Amen.